

Interpreting Spatial and Temporal Neural Activity through a Recurrent Neural Network Brain Machine Interface

Justin C. Sanchez¹, Deniz Erdogmus²,
Miguel Nicolelis³, Johan Wessberg⁴, Jose C. Principe⁵

¹Department of Pediatrics Division of Neurology, University of Florida, Gainesville, FL

²Department of Computer Science Engineering, Oregon Health and Science University, Beaverton, OR

³Department of Neurobiology, Duke University, Durham, NC

⁴Department of Physiology, Göteborg University, Göteborg, Sweden

⁵Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL

Abstract—We propose the use of optimized brain machine interface (BMI) models for interpreting the spatial and temporal neural activity generated in motor tasks. In this study, a nonlinear dynamical neural network is trained to predict the hand position of primates from neural recordings in a reaching task paradigm. We first develop a method to reveal the role attributed by the model to the sampled motor, premotor, and parietal cortices in generating hand movements. Next, using the trained model weights we derive a temporal sensitivity measure to assess how the model utilized the sampled cortices and neurons in real-time during BMI testing.

Keywords—Brain machine interface, nonlinear models, recurrent neural network, analysis of neural activity, spatio-temporal, motor systems

I. INTRODUCTION

Many brain machine interface (BMI) researchers have demonstrated the feasibility of using adaptive input-output models for reconstructing hand trajectories [1-8]. All of the proposed models in the literature demonstrated the ability to encode and store the fundamental timing relationships between neural inputs and hand trajectory [9-12]. To achieve the mapping, model parameters (weights) are adjusted to minimize the difference between the model output and hand movements using a statistical criterion such as mean-square error.

A natural next step is to analyze how the trained models extracted the spatio-temporal trends in the neural recordings. By analyzing the model parameters in a signal-processing context, we can hypothesize relationships between neurons, cortices, motor systems and behavior. This type of analysis exploits the fact that the trained model embody precise functional relationships between its inputs (the neurons, cortices), and their outputs which are a proxy for the observed behavior. Moreover, this relationship is not developed from empirical observation but solely from the data along with the model architecture.

The purpose of this study is to move beyond model performance comparisons and develop the analysis tools that may help quantify neural activity and how it is related to behavior, even if it is through an arbitrary biologically inspired, but reproducible, signal processing model. Note that analysis of neuronal interactions through trained models contrasts with traditional neuroscience methods involving

single or pair wise neuronal events (peri-event histograms, correlograms, etc.). The multi-input multi-output (MIMO) methodology proposed here is prepared to simultaneously study the interactions among large populations of neurons used in BMI experiments.

However, the success of neural analysis through signal processing models depends on the model type, its order, training assumptions and on achieving high accuracy in the mapping from neural activity to hand kinematics. Preliminary studies comparing the reproducibility of this approach through linear and nonlinear models indicated that the interpretations are consistent despite the choice of model [13].

The choice of model does have impact on the level of temporal detail that can be assigned to the neuronal activity. For BMI design, we feel that it is critically important to measure how each sampled cortex and neuron is contributing to the behavior at each moment in time. Understanding the dynamics of neural processing and how it is related to the external world will help us improve the performance of our BMI models. In this pursuit, we first derived sensitivity measures through static linear models that provided ranked lists of neurons important for the task [14]. However, this analysis is of limited value because it offers average neural importance over the training intervals. By implementing dynamic models with states, such as the recurrent multilayer perceptron (RMLP), we can derive sensitivity measures with the time resolution of the input data. In this paradigm, the use of feedback in the model can be used to relate the previous samples sensitivity to the current sample. In this study, we first train the RMLP model to predict the hand trajectory of behaving primates using various combinations of cortical activity from the primary motor, premotor and posterior parietal cortices, which provides information how the model uses the neural activity of each cortex to predict the behavior. Next, we will develop a temporal sensitivity measure that will relate at each step of the movement how the model uses the sampled cortices and neurons to track behavior.

II. METHODOLOGY

A. Model Topology

The architecture of the RMLP used in our studies (Fig. 1) consists of an input layer that accepts hundreds of

neuronal channels, a fully connected hidden layer of nonlinear processing elements (PEs), (in this case \tanh), and an output layer of linear PEs to estimate the trajectory. The RMLP uses only the instantaneous neural activity to compute each output (hand position in this case). The hidden layer consists of 5 \tanh processing elements (PEs) that are fully connected to each other with a feedback matrix (unit time delay). Each hidden PE is a nonlinear adaptive basis for the output that projects the high dimensional neuronal data. The choice of the hidden layer dimensionality was optimized to produce the best performing network with the fewest elements [10]. The state vector of the hidden layer in (1) is a nonlinear function of the linear combination of input \mathbf{x} and previous state $\mathbf{y}_1(t-1)$. The feedback of the state creates memory and allows representations on multiple timescales. We would like to note here that the feedback state representation feature of the RMLP will later enable us to derive temporal measures of neuronal activity. The output layer of the network has 3 linear PEs (for X, Y, Z coordinates) and produces the output as in (2). These projections are then linearly combined to form the outputs (position predictions) of the RMLP. The neural to motor mappings are stored in the input \mathbf{W}_1 , feedback \mathbf{W}_f , and output \mathbf{W}_2 weights and a biases ($\mathbf{b}_1, \mathbf{b}_2$).

$$\mathbf{y}_1(t) = f(\mathbf{W}_1\mathbf{x}(t) + \mathbf{W}_f\mathbf{y}_1(t-1) + \mathbf{b}_1) \quad (1)$$

$$\mathbf{y}_2(t) = \mathbf{W}_2\mathbf{y}_1(t) + \mathbf{b}_2 \quad (2)$$

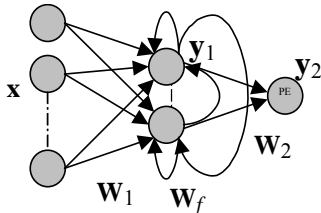


Figure 1. Fully connected, state RMLP.

B. Neural and Behavioral Data

We train the RMLP with multichannel neuronal firing times from as many as 104 cells that were collected synchronously at Duke University using two owl monkeys (*Aotus trivirgatus*). The details of the behavioral paradigm and surgical procedure for chronic microwire recordings are described elsewhere [7]. Nevertheless, we would like to briefly describe components of the paradigm that are important for this analysis. Microwire electrodes were implanted in up to four cortical regions with known motor associations [15]. Table 1 shows the assignment of the electrode arrays to the cortical regions for the two primates used in this study. The firing times of single neurons were recorded while the primates performed a 3-D reaching task that involved a right-handed reach to food and subsequent placing the food in the mouth. A stereotypical X, Y, Z trajectory plot of a single movement is given in Figure 2. Neuronal and positional data were collected during two

independent trials for each primate. Neuronal firings, binned (added) in non-overlapping windows of 100ms, were directly used as inputs to the RMLP. The primate's hand position, used as the network desired signal, was also recorded (with a time shared clock) and digitized with 200Hz sampling rate.

Table 1. Assignment of electrode arrays to cortical regions for two primates

| Primate 1 | | | |
|-------------------------|-----------------------------|-----------------------|---|
| Area 1 33 cells | Area 2 21 cells | Area 3 27 cells | Area 4 23 cells |
| Posterior Parietal (PP) | Primary Motor (M1, array 1) | Dorsal Premotor (PMd) | Primary Motor & Dorsal Premotor (M1/PMd, array 2) |

| Primate 2 | |
|--------------------|-----------------------|
| Area 2 37 cells | Area 3 17 cells |
| Primary Motor (M1) | Dorsal Premotor (PMd) |

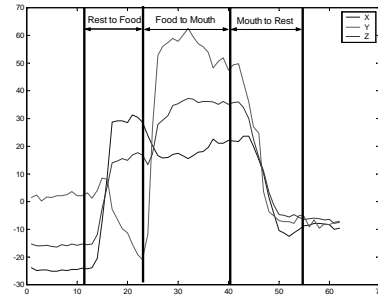


Figure 2. Stereotypical reaching movement segmented into rest/food, food/mouth, and mouth/rest motions.

C. Model Training

Optimal weights ($\mathbf{W}_f, \mathbf{W}_1, \mathbf{W}_2$ and the vectors \mathbf{b}_1 and \mathbf{b}_2) that define the neural to motor transfer function are determined by minimizing the mean squared error (MSE) using backpropagation through time (BPTT) [10, 16, 17]. A trajectory of 30 samples was chosen to approximately match the duration of a reaching movement and learning rates of 0.01, 0.01, and 0.001 for the input, feedback, and output layers respectively were used in the NeuroSolutions software package [18]. Momentum learning was also implemented with a rate of 0.7. One hundred Monte Carlo simulations with different initial conditions were conducted with 20,010 consecutive bins (2,001 secs) of neuronal data to improve the chances of obtaining the global optimum. Of all the Monte Carlo simulations, the network with the smallest error achieved a MSE of 0.0203 ± 0.0009 . A small training standard deviation indicates the network repeatedly achieved the same level of performance. Training was

stopped using the method of cross-validation (batch size of 1000 pts.) to maximize the generalization of the network [19].

In testing, the network parameters were fixed and 3,000 consecutive bins (300 secs) of novel neuronal data were fed into the network to predict new hand trajectories. Fig. 3 shows the output of one of the networks (primate 1) in the test set with 3-D hand position decomposed into X, Y, and Z coordinates. We can see from the plots that the RMLP repeatedly produces outputs that mimic the hand trajectories. We also conclude that during the short period of observation (5 min), there is no noticeable degradation of the model fitting across time. The traditional way to report results in BMI is through the correlation coefficient (CC) computed between the model output and the actual hand trajectory. In Table 2 we present the testing CC values for both primates. The testing CC values obtained here are consistent with those found in the BMI literature [1-8].

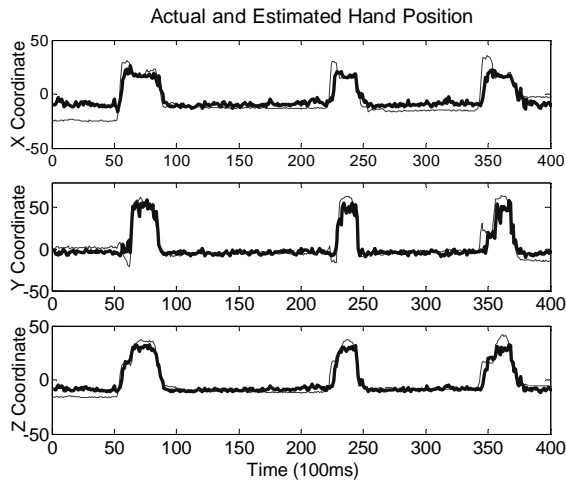


Figure 3. Testing trajectories for primate 1 (bold – model, thin – actual hand position).

Table 2. Testing correlation coefficients for the two subjects

| | Performance (CC) | | |
|-----------|------------------|-----------|-----------|
| | X | Y | Z |
| Primate 1 | 0.53±0.33 | 0.71±0.27 | 0.71±0.38 |
| Primate 2 | 0.58±0.21 | 0.50±0.18 | 0.77±0.12 |

III. CORTICAL CONTRIBUTIONS

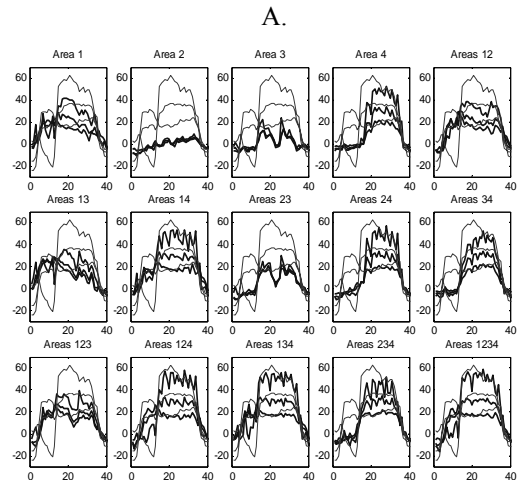
We are interested in investigating which cortical regions the RMLP uses to reconstruct the triad of movements defined in Fig. 2. Each reaching movement is segmented to three reaches: rest/food, food/mouth, and mouth/rest. In typical BMI experiments, since the goal is to produce the best reconstruction of the trajectory, researchers typically utilize all of the sampled neurons. However, since we are interested in understanding how the activity of each cortex is used by the model to reconstruct the movement, we are

going to choose different cortices to train the RMLP, and observing the network outputs, we can build a set of hypothesis about the importance of each cortex and compare with established neurophysiologic principles.

Using the methods described in **IIC**, we trained 15 RMLPs for primate 1’s data and 3 RMLPs for primate 2’s data. To visualize how the cortices contribute, the X, Y, Z network outputs (bold) and the actual hand coordinates for one sample movement are plotted in Fig. 4 for each network and primate. For simplicity we have plotted only one movement however detailed analysis of all the testing movements provided similar results for each primate. The title of each plot indicates the cortices used as model inputs. For example, title Area 123 indicates that PP, M1, and PMd were used as inputs (see Table 1).

For the model trained with primate 1, area 1 produced an increase in amplitude during rest/food, but showed a gradual decrease in food/mouth. Primate 2 was not implanted in area 1 and subsequently all models did not produce increases in trajectory during the reach to food. The model trained with area 2 for primate 1 does not display any correlation to this desired trajectory even though neuronal firing in this region is nonzero. Models trained with area 4 of primate 1 and area 2 of primate 2 produce similar trajectories during the food/mouth segment. It is interesting that for primate 1 areas 2 and 4 had a similar number of neurons and contained samples from M1 but model testing produced different trajectories. Sharp changes in the model output appear in movement transitions for the networks trained with Area 3 in both primates. Both the WF and RMLP display the following trends in the hand trajectory reconstruction:

- Models require the PP cortex to reproduce rest/food trajectories.
- For primate 1, the array 2 M1 neuronal activity is not used to create food/mouth trajectory.
- Models use the PMd for transitions in trajectory.
- Models use M1 neuronal activity (Primate 1 area 4, Primate 2 area 2) to code similar movements.



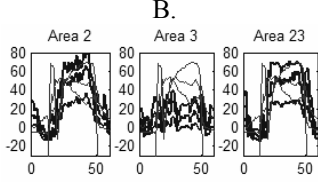


Figure 4. Testing output X, Y, and Z trajectories (bold) for one desired movement (thin) from fifteen RMLPs trained with neuronal firing counts from all combinations of four cortical areas. A) Primate 1. B) Primate 2.

IV. TEMPORAL SENSITIVITY MEASURES

For BMI design, the ability to assess the contribution of cortices and neurons in real-time for a variety of behaviors is necessary to uncover the unknown aspects of functional motor systems, measure the plasticity of closed-loop neuroprosthetic control, and to provide additional information on decoding algorithm design.

The procedure for deriving the sensitivity for a feedforward topology is an application of the chain rule [20] and produces a static measure of how modulations in the input (neuronal activity) produce changes in its output, which is a proxy for behavior if the mapping is accurate. In (3) we present the chain rule for a single hidden layer neural network.

$$\frac{\partial y_2}{\partial x} = \frac{\partial y_2}{\partial y_1} \frac{\partial y_1}{\partial x} \quad (3)$$

In the RMLP, a sensitivity analysis based on the Jacobian of the output vector with respect to the input vector extends this analysis to the sample rate of the input. Since the RMLP model displays dependencies over time that result from feedback in the hidden layer, the partial derivative of y_1 with respect to x must be modified to include temporal information. Starting at each time t , the sensitivities to instantaneous inputs as well as previous inputs are computed. In the RMLP, the chain rule is applied as shown in (4)-(7), where \mathbf{D}_t is the derivative of the hidden layer nonlinearity evaluated at the operating point shown in (5) using the input sample at time t . Notice at $t = 0$ there are no dependencies on \mathbf{y}_1 . If time is clocked back one cycle, the dependencies introduced by the feedback must be included, which is shown in (6). At each clock cycle back in time, an additional $\mathbf{W}_f^T \mathbf{D}_{t-i}$ is multiplied to obtain the general form in (7). Experimentally we determined that the effect of an input decays to zero over a window of 20 samples as shown in Fig. 5. At each time t the absolute-sensitivity of the output with respect to the inputs is represented as the averages of the absolute values of the sensitivities over the 20-sample. The procedure described in (4)-(7) will produce a matrix of values of dimension (#Neurons x #Coordinates x Time). Note that nowhere in this derivation the desired response is used, just the output of the model, but if the error is small a reasonable approximation of the sensitivity to behavior can be estimated.

$$\frac{\partial \mathbf{y}_2(t)}{\partial \mathbf{x}(t)} = \mathbf{W}_2^T \mathbf{D}_t \mathbf{W}_1^T \quad (4)$$

$$\mathbf{D} = \text{diag}(f'(z_1^1) \quad f'(z_1^2) \quad \dots \quad f'(z_1^n)) \quad (5)$$

$$\frac{\partial \mathbf{y}_2(t)}{\partial \mathbf{x}(t-1)} = \mathbf{W}_2^T \mathbf{D}_t \mathbf{W}_f^T \mathbf{D}_{t-1} \mathbf{W}_1^T \quad (6)$$

$$\frac{\partial \mathbf{y}_2(t)}{\partial \mathbf{x}(t-\Delta)} = \mathbf{W}_2^T \mathbf{D}_t \left(\prod_{i=1}^{\Delta} \mathbf{W}_f^T \mathbf{D}_{t-i} \right) \mathbf{W}_1^T \quad (7)$$

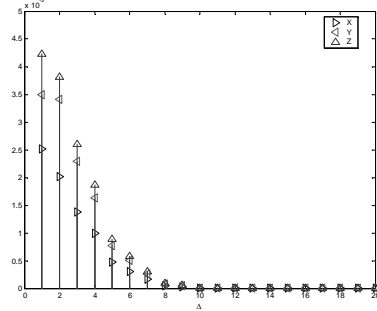


Figure 5. a) Sensitivity at time t for a typical neuron as a function of Δ .

The temporal sensitivity analysis can be used to estimate multiscale cortical neural ensemble organization using the trained RMLP as a proxy. At the most macroscopic level we can ask the question ‘‘How do modulations in the neuronal input affect each of the three movement coordinate directions?’’ This question can be addressed by working with the RMLP and summing the temporal sensitivity matrix over the neural ensemble to yield the time-series shown in Fig. 6 subplots 1. For each of the primates, the temporal sensitivity is time synchronized with a representative movement (bottom most subplot). For simplicity, we present only one reaching movement for each primate. We can see that the neural contributions to each direction are markedly different for the models trained with each of the primate’s data; primate 1 model produces large sensitivities during food/mouth for coordinates Y and Z while primate 2’s model does not. Additionally, neuronal coordinate modulation produced by primate 2 is delayed 1 second after the movement compared to primate 1.

At the mesoscopic level, we can sum the RMLP temporal sensitivity matrix down to each cortical area and produce the time-series shown in Fig. 6 subplots 2. Over the time course, we can see that the model assigns to each cortex a varying contribution w.r.t. behavior depending on the movement type (e.g. rest/food, food/mouth, velocity, and constant position). Comparatively, primate 1’s RMLP is producing a higher level of movement detail compared to the model trained with primate 2 because we observe modulations of sensitivity during the movement. Primate 2’s RMLP modulates up and remains constant indicated by a constant low sensitivity. This quantification of the temporal

sensitivity relates directly to many of the qualitative observations expressed in III. For example, primate 1 Area 2 (M1) temporal model sensitivity is the lowest compared to the other cortices indicating that its neuronal modulation provides the least contribution to the output. On the other hand, area 2 (M1) for primate 2 model produced much higher temporal sensitivities than area 3 indicating it is more important for producing the movement.

At the microscopic level, we can ask which neurons are affecting the output of the model the most. Fig. 6 subplots 3 depicts the neuronal temporal sensitivities for area 4 (primate 1) and area 2 primate 2). For primate 1 we can see that the model uses the strong modulation of a single neuron’s activity during the food/mouth trajectory. Thirteen out of the thirty-seven neurons in area 2 are modulating strongly at the beginning and end of the entire movement and are used in primate 2 model. For each of these animal models, the temporal sensitivity measure is indicating two different neuronal distributions of representation in each cortex.

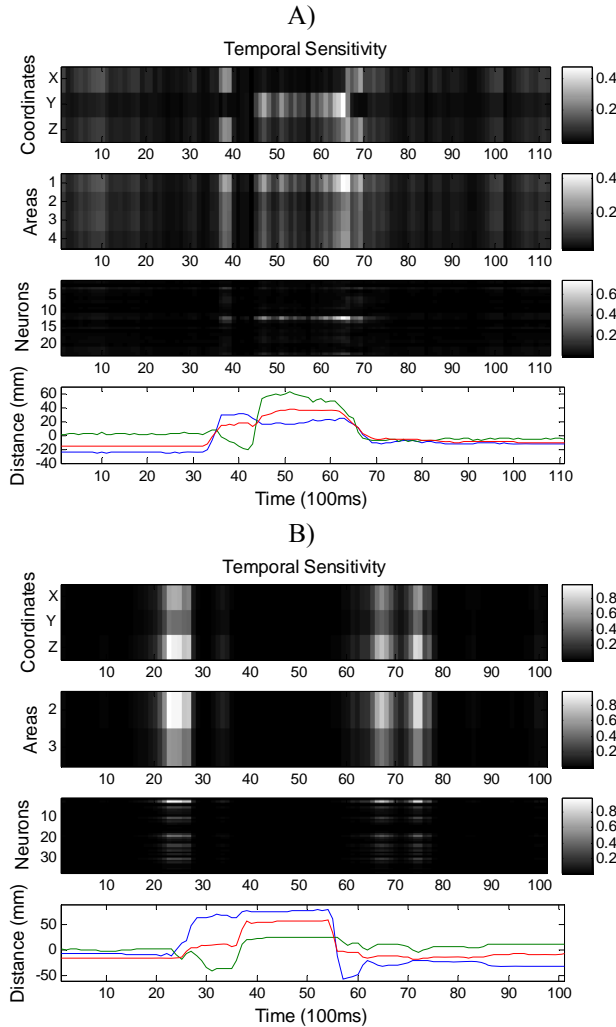


Figure 6. Temporal sensitivities for Cartesian coordinates, neurons and cortices. A) Primate 1, B) Primate 2.

V. DISCUSSION

In BMI experiments, interpreting the relationship between ensemble neural activity and behavior is a daunting task due to the large dimensionality and diversity of the cortical networks that are probed. Moreover, from a functional neurophysiology and systems neuroscience perspective the goal of *simultaneously* studying neuronal interactions to understand motor system invokes a need for new spatio-temporal analysis techniques. We presented here tools for interpreting neural data through optimally trained MIMO models. The critical step in such a task was choosing an appropriate model for incorporating both the spatial and temporal aspects. The RMLP offered just that: multiple inputs, dynamics, and nonlinearity.

It is tempting to equate directly the learned input output model dependencies with causal relationships between neural data and behavior, but we must remember that this is achieved through a signal processing model, which makes the interpretation dependent upon the model type, order, training constraints and accuracy. However, this procedure opens a quantitative, high resolution approach to study neural population activity, which may corroborate or raise new hypothesis about mesoscopic cortical activity and how it might create behavior. After our cautious presentation, we will present below the implications of the analysis. Our first comment is related to the neurophysiological roles of the motor, premotor, and parietal cortices. Two themes hold true between the subjects. The PP was necessary for reconstructing the reach to the food and the M1 (area 4 primate 1, area 2 primate 2) was necessary for reconstructing the reach from food to mouth. In terms of BMI design, this indicates that the electrodes should be placed strategically throughout the motor cortex to capture vital information. It is still unclear if a more widespread sampling of M1 will provide the information or if PP is truly necessary to capture the initial segment of the reach. Even within M1, we observed diversity in the contributions of neurons from each array (see array 2 vs array 4, primate 1). If electrodes are not placed in a cortical region important for a part of the movement, the trajectory is not guaranteed to be reconstructed well. Nevertheless, we can compare our observations with those found in experimental neuroscience which has linked the posterior parietal cortex to motor imagery [21], visual/tactile manipulation of objects [22], and spatial coordinate transformations [23].

Our second comment regards the schemes of neural coding employed by each of the animals. Multiscale temporal sensitivity analysis revealed that each animal is coding the movement differently at each scale. Over the movement time span, the computed sensitivities had preferentially high values for specific Cartesian coordinates, cortices, and neurons for each animal. The primary difference is that primate 2 modulated up and down its neuronal activity only at the beginning and end of the trajectory while primate 1 had modulations throughout. This

measurable difference in neural coding translates directly in differences in the detail of trajectory reconstruction (see Fig 4A(areas 1234) and 4B(areas23)). While the animals in general performed the same reaching task, the trajectories visited by each of the animals were not identical. Since in BMI design we cannot expect that all reaching movements are the same, we need to address how models are going to address the diversity of natural reaching movements. The degrees of freedom of neural activity seem to be limited to local organization and modulation for this class of signal processing models. Due to the different importance of neurons in the various segments of the reaching task, we anticipate that important neurons will vary for different movements and individuals. If this is the case, two problems arise. First, a given sampling of neurons may provide a limited repertoire of reconstructable movements. Second, a network trained for all possible movements may perform worse than one trained just for one movement. This is particularly true for linear mappers, but it will affect to a certain extent nonlinear models.

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